



**PATNA UNIVERSITY
OF SCIENCE AND TECHNOLOGY**

FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES

SCHOOL OF NATURAL AND APPLIED SCIENCES

DEPARTMENT OF BIOLOGY, CHEMISTRY AND PHYSICS

QUALIFICATION : BACHELOR OF SCIENCE	
QUALIFICATION CODE: 07BOSC	LEVEL: 7
COURSE CODE: MMP701S	COURSE NAME: MATHEMATICAL METHODS IN PHYSICS
SESSION: JULY 2023	PAPER: THEORY
DURATION: 3 HOURS	MARKS: 100

SUPPLEMENTARY/SECOND OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER(S)	Prof Dipti Ranjan Sahu
MODERATOR:	Prof. S. C. Ray

INSTRUCTIONS
1. Answer ALL the questions. 2. Write clearly and neatly. 3. Number the answers clearly.

PERMISSIBLE MATERIALS

Non-programmable Calculators

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

Question 1**[25]**

- 1.1 The law of decay states that the rate of decay for a radioactive material is proportional to the number of atoms present.
- 1.1.1 Formulate the differential equation and determine the amount of radioactive material left at any time, t by solving the differential equation. (5)
- 1.1.2 Determine the half-life of a radioactive material using solution of differential equation. (5)
- 1.1.3 In two years, 3 g of a radioisotope decay to 0.9 g. Determine both the half-life T and the decay rate k . (5)
- 1.2 Solve the equation,
$$\frac{dx}{dt} + t^2x = Cost$$
 (5)
- 1.3 Solve the differential equation $(2xy-3x^2) dx + (x^2-2y) dy = 0$ (5)

Question 2**[25]**

- 2.1 Suppose that a car is going 76 m/s when brakes are applied at $t = 2$ s. Suppose that the nonconstant deceleration is known to be $a = -12t^2$. Formulate the differential equation and determine the distance the car travels. (10)
- 2.2 Find the particular solution of $x' + x = e^{-t}$ (10)
- 2.3 Solve the equation: $5y'' + 2y' + 2y = 0$. (5)

Question 3**[25]**

- 3.1 Find the eigenvalues and eigenvector of the matrix A given by (10)

$$A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

- 3.2 Solve the following system of equations using Gauss-Jordan Elimination: (10)
 $-3x - 2y + 4z = 9$
 $3y - 2z = 5$
 $4x - 3y + 2z = 7$
- 3.3 If $[2x \ 3] \begin{bmatrix} 1 & 2 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ 8 \end{bmatrix} = 0$, find the value of x (5)

Question 4

[25]

- 4.1 Let v be a vector in an inner product space V over \mathbb{R} .
Suppose that $\{u_1, \dots, u_n\}$ is an orthonormal basis of V .
Let θ_i be the angle between v and u_i for $i=1, \dots, n$. Prove that $\cos^2\theta_1 + \dots + \cos^2\theta_n = 1$ (10)

- 4.2 Verify if the vectors $V_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}; V_2 = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}; V_3 = \begin{pmatrix} 0 \\ 3 \\ 5 \end{pmatrix}$ are linearly independent. (5)

- 4.3 Express first two Legendre Polynomials $P_0(x)$ and $P_1(x)$ using the given function (4)

$$P_n(x) = \frac{(2n)!}{2^n (n!)^2} \left[x^n - \frac{n(n-1)}{2(2n-1)} x^{n-2} + \frac{n(n-1)(n-2)(n-3)}{2 \times 4(2n-1)(2n-3)} x^{n-4} - \dots \right]$$

- 4.4 Using the Laplace transform find the solution for the following equation (6)

$$\frac{\partial y(t)}{\partial t} = e^{-3t}$$

with initial conditions $y(0) = 4$ and $Dy(0) = 0$

..... END.....